Unit Zero

Name: Key

This packet is due, in its entirety, by **MONDAY DECEMBER 2ND**. Following the completion of this packet you will be required to take a unit assessment, similar to that given at the beginning of the year.

Concept 1: ADDING AND SUBTRACTING POLYNOMIALS.

In order to add and subtract polynomials we have to understand what LIKE TERMS are. LIKE TERMS are terms that have the same variables and powers. The coefficients do not need to match.

For example:

- 3x and -5x are like terms because both terms have the variable x, raised to the same power (one). We can combine these like terms by combining their coefficients. 3x combined with -5x would be -2x.
- 3x and 4y are not like terms because they deal with two different variables (x and y) so we can not combine them.
- 3x and $-5x^2$ are not like terms because, although they deal with the same variable of x, one is raised to the power of one and the other to the power of two. We can not combine these two terms.

ADDING POLYNOMIALS.

- To add two or more polynomials together we simply combine any like terms. Be sure to take into account the sign of your coefficients.

Example: $(2x^2 + 1) + (x^2 - 2x + 1)$ Here we can see $2x^2$ and x^2 as like terms, together they combine to $3x^2$ (notice we do not add their exponents, just their coefficients). -2x does not have a like term so remains unchanged. We have two constants (+1 and +1) that together combine to +2. The sum of these two polynomials would be: $3x^2-2x+2$

SUBTRACTING POLYNOMIALS

- When subtracting polynomials we have to be careful of when we are subtracting a negative. The action of subtracting a negative is the same as adding. Here is an example:

Example: $3(x^2 - 2x + 3) - 4(4x + 1) - (3x^2 - 2x)$

First we distribute: $3x^2 - 6x + 9 - 16x - 4 - 3x^2 + 2x$

Then we combine like terms: $-20x + 5$

Adding and Subtracting Polynomials

Perform the operations.

1. $(12y^2 + 17y - 4) + (9y^2 - 13y + 3) =$ $21y^2 + 4y - 1$

2. $(2x^3 + 7x^2 + x) + (2x^2 - 4x - 12) =$ $2x^3 + 9x^2 - 3x - 12$

3. $(-3m^2 + m) + (4m^2 + 6m) =$ $m^2 + 7m$

4. $(7z^3 + 4z - 1) + (2z^2 - 6z + 2) =$ $7z^3 + 2z^2 - 2z + 1$

5. $(3a^2 + 2a - 2) - (a^2 - 3a + 7) =$ $2a^2 + 5a - 9$

6. $(5x^2 - 2x - 1) - (3x^2 - 5x + 7) =$ $2x^2 + 3x - 8$

7. $-(3z^2 + 4z) - (6z^2 - 2) =$ $-9z^2 - 4z + 2$

$-3z^2 - 4z - 6z^2 + 2$

Concept 2: MULTIPLYING BINOMIALS

Unlike adding and subtracting polynomials, when multiplying we do not have to be concerned with like terms, we can multiply any two terms together, regardless of whether they are like terms.

Multiplying binomials requires us to understand the distributive property (which you used on the previous page). However, instead of distributing one term to several others we are distributing two terms.

Example: $(4x - 5)(-2x + 7)$

1. We distribute -5 to -2x and 7, giving us $10x - 35$
2. Then we distribute 4x to -2x and 7, giving us $-8x^2 + 28x$
3. Then we combine these two together: $10x - 35 - 8x^2 + 28x = -8x^2 + 38x - 35$

1) $(3n+2)(n+3)$

$2n+6+3n^2+9n$

$\boxed{3n^2+11n+6}$

2) $(n-1)(2n-2)$

$-2n+2+2n^2-2n$

$\boxed{2n^2-4n+2}$

3) $(2x+3)(2x-3)$

$6x-9+4x^2-6x$

$\boxed{4x^2-9}$

4) $(r+1)(r-3)$

$r-3+r^2-3r$

$\boxed{r^2-2r-3}$

5) $(2n+3)(2n+1)$

$6n+3+4n^2+2n$

$\boxed{4n^2+8n+3}$

6) $(3p-3)(p-1)$

$-3p+3+3p^2-3p$

$\boxed{3p^2-6p+3}$

7) $(3p+3)(3p+2)$

$9p+6+9p^2+6p$

$\boxed{9p^2+15p+6}$

8) $(k-2)(k-3)$

$-2K+6+K^2-3K$

$\boxed{K^2-5K+6}$

9) $(v-1)(3v-3)$

$-3v+3+3v^2-3v$

$\boxed{3v^2-6v+3}$

10) $(2x-3)(3x+3)$

$-9x-9+6x^2+6x$

$\boxed{6x^2-3x-9}$

11. A rectangle has a length of $3x-7$ and a width of $-2x+4$. Write an expression for the rectangle's perimeter and an expression for the area.

$(3x-7)(-2x+4)$

$14x-28+6x^2+12x$

Perimeter: $2x-6$

Area: $6x^2+26x-28$

12. A rectangle has a length that is four more than triple its width. Write an expression for the rectangle's perimeter and an expression for the area.

$L=3w+4$

Perimeter: $8w+8$

Area: $3w^2+4w$

$w(3w+4)=3w^2+4w$

13. A rectangle has a perimeter of $24x + 52$ and a length of $4x + 3$. Find the width of this rectangle and then use that width to write an expression for the area.

$(4x+3)(8x+23)$

$24x+69+32x^2+92$

$$\begin{array}{r} 24x+52 \\ -\ \ 8x+6 \\ \hline 16x+46 \end{array} \rightarrow \text{width} = 8x+23$$

Width: $8x+23$

Area: $32x^2+116x+69$

14. A triangle has three sides (Side A, Side B, and Side C). Side A is five less than twice the length of Side B. Side C is four times as long as sice B. Write an expression that would represent the perimeter of this triangle.

$A = 2b-5$

$b = b$

$C = 4b$

$2b-5+b+4b$

$\boxed{7b-5}$

Concept 3: SIMPLIFYING RADICALS

To simplify radicals we have to understand what a **PERFECT SQUARE** is and what a **PRIME NUMBER** is:

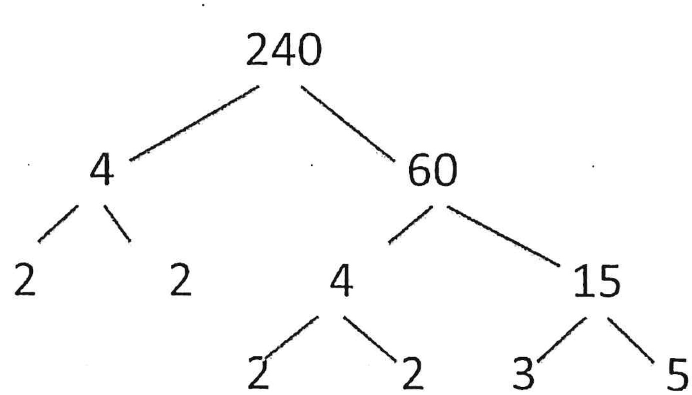
- **A PERFECT SQUARE** is the product of a rational number multiplied by itself. Common perfect squares are 4 (2x2) ; 9 (3x3); 16 (4x4); 25 (5x5) and so on.

- **A PRIME NUMBER** is a whole number greater than 1 whose only factors are 1 and itself. Common prime numbers are 2, 3, 5, 7, 11, 13, 17.. Etc.

To simplify a radical we use something called PRIME FACTORIZATION which is a process of breaking down a number into its prime factors.

Example: $\sqrt{240}$

Taking a look at our prime factors, we want to take note of factors that come in **pairs**. By doing this we are finding perfect squares within the factors of 240.



Each group of two prime numbers gets "pulled out" to the outside of the radical sign and becomes our coefficient..

If there is more than one pair of numbers (for example we have 2 groups of 2 in our prime factors above) our coefficient of the simplified radical would be the product of our pairs (int his case, 4)

Any prime numbers left unpaired get multiplied together, this product becomes our new radicand (what is left inside the radical symbol).

$\sqrt{240} = 4\sqrt{15}$

We can follow this same process when dealing with variables in our radicand

1) $\sqrt{147m^3n^3}$

$7mn\sqrt{mn}$

2) $\sqrt{200m^4n}$

$10m^2\sqrt{n}$

3) $-4\sqrt{192x}$

$-32\sqrt{3x}$

4) $2\sqrt{8p^2q^3r}$

$4pq\sqrt{qr}$

5) $\sqrt{6} + \sqrt{24}$

$3\sqrt{6}$

6) $-\sqrt{3} + 3\sqrt{3}$

$2\sqrt{3}$

7) $3\sqrt{54} + 2\sqrt{24}$

$13\sqrt{6}$

8) $\sqrt{25} + 2\sqrt{81} - \sqrt{36}$

17

9) $-2\sqrt{12}-\sqrt{12}$

$-6\sqrt{3}$

10) $3\sqrt{12}-2\sqrt{12}-\sqrt{54}$

$-3\sqrt{6}+2\sqrt{3}$

11) $\sqrt{90}-\sqrt{40}$

$\sqrt{10}$

12) $\sqrt{48}-\sqrt{3}-\sqrt{2}$

$3\sqrt{3}-\sqrt{2}$

13) $2\sqrt{3}\bullet 4\sqrt{3}$

24

14) $-3\sqrt{15}(5+\sqrt{3})$

$-15\sqrt{15}-9\sqrt{5}$

15) $(4\sqrt{5}-3)(\sqrt{5}-2)$

$-11\sqrt{5}+26$

16) $2\sqrt{6}(4-\sqrt{8})+\sqrt{3}(\sqrt{27}+7)$

$8\sqrt{6}-8\sqrt{3}+7\sqrt{3}+9$

$\boxed{8\sqrt{6}-\sqrt{3}+9}$

Concept 4: Operations with rational and irrational numbers.

Rules:

- A. The sum of two rational numbers is always rational.
- B. The sum of two irrational numbers is sometimes irrational.
- C. The sum of two irrational numbers is sometimes rational.
- D. The sum of one rational number and one irrational number is always irrational.
- E. The product of two rational numbers is always rational.
- F. The product of two irrational numbers is sometimes rational
- G. The product of two irrational numbers is sometimes irrational
- H. The product of one rational number and one irrational number is sometimes irrational.

A **RATIONAL NUMBER** is a number that can be expressed as a fraction

An **IRRATIONAL NUMBER** is a number that cannot be expressed as a fraction for any integers and . Irrational numbers have decimal expansions that neither terminate nor become periodic (do not repeat in a pattern)

Examples of irrational numbers: **π**, square root of any non-perfect square, 2.678954...

Identify whether each will be rational or irrational. Then state which rule you used from above.

1) $0 \cdot \pi$ rational

2) $\sqrt{3} \cdot \frac{1}{\sqrt{3}}$ rational

3) $\sqrt{5} - \sqrt{5}$ rational

4) $2 \cdot \sqrt{4}$ rational

5) $\pi + \pi$ irrational

6) $\sqrt{5} + \sqrt{5}$ irrational

Concept 5: Dimensional Analysis: you will need to use all conversion charts you received in class. See your teacher if you need additional copies of these.

1. How many seconds are in a year?

31,536,000 seconds per year.

2. The circumference of Earth is 25,000 miles. What is the circumference in cm?

4.02336×10^9

3. To reach the recommended daily intake of calcium, a person must drink 600. mL of milk a day. If 200. mL of milk contains 300. mg of calcium, how much calcium, in kg, is a person recommended to intake in 30 days?

$\frac{600}{200} = 3 \longrightarrow 3(300) = 900$ mg of ~~calcium~~/day

$900(30) = 27{,}000$ mg of calcium

4. There are 2600. miles between Boston and Los Angeles (in a straight line). If a plane flies at 600. miles/hour. How long is the flight between Boston and LA?

$4\frac{1}{3}$ hours.

5. If you earned one penny for every 10 seconds of your life then how many dollars would you have after 65 years?

$ 20,498.40

6. A car's gas tank holds 12 gallons and is ¼ full. The car gets 20. miles/gallon. You see a sign saying "Next gas 82 miles". Can you make it to the gas station before running out of gas?

12(20) = 240 miles = full tank

$\frac{1}{4}(240)$ = 60 miles

no you will only make it 60 miles with $\frac{1}{4}$ of a tank.

7. A runner competed in a 5-mile run. How many yards did she run?

8,800 yards

8. In the Tour de France, cyclists ride 3,653.6 km over 20 days. How many miles do they go?

2,270.24 miles

9. In the US milk is sold by the gallon, while in Italy it is sold by the liter. How many liters of milk would you need to equal one gallon? (1 L = 1.06 qts; 4 qts = 1 gal)

About 3.77 liters.

10. If you go to school for 180 days each year and each day is 7 hours long, how many minutes are spent in school in one year?

180(7) = 1,260 hours

1,260 (60) = 75,600 minutes