Section 5.3 Notes: More Theoretical Probability Rules

Conditional Probability

Sometimes the knowledge that one event has occurred changes the probability that another event will occur.

- The probability of being in a car accident increases if you know that it is raining outside
- Suppose that the pass rate on the AP is exam is 80%. That is, for a randomly selected student, P(pass) = .80. However, if you know that the student got a B in the class, then the probability increases 95%. That is, for a randomly selected student, P(pass given that you have a B) = .95.
    - Notation: P(pass | B) = .95

The above scenarios illustrate what we call **Conditional Probability.**

Conditional Probability:

Calculating Conditional Probabilities: There are 2 ways…

1. Use common sense (when a table is given)
2. Use the formula:

Conditional Probability

Independent/Dependent Events – The Multiplication Rule

Independent Events **vs.** **Dependent Events**

Say we have two events (A and B), and we want to know the probability that both events occur. Then we want to know the probability of event A **AND** event B occurring. Here's how we find it:

The Multiplication Rule:

For two events A and B, the probability of A **AND** B occurring is…

The AP Statistics Formula Page!
This is what you are given:

(II) Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Examples:

1. **Cards.** You draw a card at random from a standard deck of 52 cards. Find each of the following conditional probabilities.

   a) The card is a heart, given that it is red.

   $P(H|R) = \frac{P(H \cap R)}{P(R)} = \frac{(13/52)}{0.5} = 0.5$

   b) The card is red, given that it is a heart.

   $P(R|H) = \frac{P(R \cap H)}{P(H)} = \frac{0.25}{0.25} = 1$

   c) The card is an ace, given that it is red.

   $P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{(2/52)}{0.5} = 0.077$

   d) The card is a queen, given that it is a face card.

   $P(Q|F) = \frac{P(Q \cap F)}{P(F)} = \frac{4}{12} = 0.33$

2. A researcher interested in eye color versus success in a math program collected the following data from a random sample of 2000 high school students.

	brown	blue	Total
fail	190	10	200
pass	1710	90	1800
Total	1900	100	2000

   a) What is the probability that a student from this group fails the math program?

   $200/2000 = 0.1$

   b) What is the probability that a student from this group fails the math program given that he/she has blue eyes?

   $P(F|BL) = \frac{10}{100} = 0.1$

   c) Are blue eyes and failing the math program independent or dependent? Justify your response.

   $P(BL) = 100/2000$
   $P(Fail) = 200/2000$

   $P(BL) = P(BL|F)$
   $100/2000 = \frac{10}{200} =$

   yes $P(BL) = P(BL|F)$ they R indep.

   $P(Fail) = 200/2000$
   $P(Fail|BL) = \frac{10}{100}$ = Independent

3. Recall this example:

Ice cream preference

Age		Likes Chocolate Ice Cream	Likes Vanilla Ice Cream	Likes Strawberry Ice Cream	Total
Age	Under 30	604	366	322	1292
	30-51	404	424	286	1114
	Over 51	336	72	386	794
	Total	1344	862	994	3200

Are liking chocolate ice cream and being over 51 independent? Justify your answer.

More with Conditional Probabilities:

Occasionally you will be a given a problem that gives you several probabilities, both probabilities of single and compound events. It will then ask you to find another probability using only the given information. If this is the case, draw and label a TREE DIAGRAM to help you out.

Example: Luggage. Leah is flying from Boston to Denver with a connection in Chicago. The probability her first flight leaves on time is 0.15. If the flight is on time, the probability that her luggage will make the connecting flight in Chicago is 0.95, but if the first flight is delayed, the probability that the luggage will make it is only 0.65.

a. Are the first flight leaving on time and the luggage making the connection independent events? Explain.

b. What is the probability that her luggage arrives in Denver with her?

c. Suppose you pick her up at the Denver airport, and her luggage is not there. What is the probability that Leah's first flight was delayed?

Example 2: Vote for Dole. In 1999, Elizabeth Dole was a candidate to become the first woman president in US history, and many observers assumed that she would have particular strength among women. According to a Gallup poll, "She did slightly better among Republican women than among Republican men, but this strength was not nearly enough to enable her to challenge Bush. In the October poll, Dole received the vote of 16% of Republican women, compared to 7% of Republican men." [Source: www.gallup.com (2000).] With the additional information that the Republican party is about 60% male, find the probability that a Republican randomly selected from the October survey would have voted for Dole.

Review

Practice Problems

Test Monday!