Section 5.2 Notes: Probability Models

Recall:

Experimental Probability	vs.	**Theoretical Probability**
Calculate from data w/experiment OR simulation		Calculate w/rules & formulas

Chance behavior is unpredictable in the ***short run***, but has a regular and predictable pattern in the ***long run***. For example, toss a coin –in the long run, about ½ will be heads P(heads) = ½.

Note: Probability can be written as decimal Fraction OR %

Myths and Common Misinterpretations of Probabilities:

- Short-run regularity
  - Remember that the patterns of probability apply to the long run.
- The Law of Averages
  - Usually, random events are independent of previous events. For instance just because a fair coin has been flipped heads six times in a row, it is NOT more likely that the seventh flip will be tails. This is similar to what's called the *Gambler's Fallacy*.

Theoretical Probability Rules

Basic Vocabulary:

- **Sample Space –** Set of all possible outcomes (denoted S)
- **Event –** subset of the sample space

Examples: What is the sample space in each of these situations?

a) Toss a coin once {heads, tails}

b) Toss a coin 2 times {HH, TT, HT, TH}

c) Roll 1 die {1, 2, 3, 4, 5, 6}

d) Choose 1 student from this class

{Sydney A, Jeffrey F, James P, Cristinas, Annikas, Bella T}

Fundamental Counting Principle A.K.A Multiplication Principle:

If you can do 1 task in "a" ways and another "b" ways then both tasks can be done a(b) ways

Examples:

1) If you go to a pizzeria and they have 3 different types of crusts, (thin, Sicilian, stuffed) 7 different toppings (pepperoni, sausage, onions, BBQ chicken, bacon, salami, mushrooms) and 5 different beverages (water, soda, lemonade, HI-C, and beer), how many different combinations of one crust, one topping and one beverage can you have? $3 \cdot 7 \cdot 5 = 105$ ways

2) The standard Connecticut license plate is three letters, followed by 3 numbers. How many license plates can you form using these specifications? $\underline{26} \cdot \underline{26} \cdot \underline{26} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} = 17{,}576{,}000$

3) How many 5 digit zip codes are possible? $\underline{10}\ \underline{10}\ \underline{10}\ \underline{10}\ \underline{10}$ OR $10^5 = 100{,}000$

4) John and Kate Gosselin had 8 children. The order from oldest to youngest is GGGGBBGB. How many different gender outcomes are possible when having 8 children? $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$ OR $2^8 = 256$

Probability Facts:

1. Prob always b/t 0 & 1 (0% 100%)
$0 \le P(A) \le 1$

2. Sum of all probabilities for all outcomes = 1
$P(S) = 1$

3. Prob that an event does not occur is called a complement: 1 − prob event happening $P(A^c) = 1 - P(A)$

4. Prob at least one event occurs is 1 − prob that event never occurs
$P(\text{at least one}) = 1 - P(\text{none})$

Probability Notation:

P(A) = prob. of event A occuring

$\cup$ union
add (OR)

$\cap$ Intersection
(AND)

P(A∪B)
Prob of A or B happening

P(A∩B)
Prob of A and B happening

Mutually Exclusive/Disjoint Events – The Addition Rule

- What are the chances of getting into Duke OR Stanford?
- What's the probability of rolling a 6 OR flipping a heads?
- What proportion of students like ice cream OR cookies?

Cannot be in 1st OR 2nd period @ same time

All of the questions above are asking for a probability. What else do they have in common? OR

Say we have two events (A and B), and we want to know the probability that either event occurs. Then we want to know the probability of event A **OR** event B occurring. Here's how we find it:

The Addition Rule:

For two events A and B, the probability of A **OR** B occurring is…

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

OR

Mutually Exclusive vs. ~~Disjoint~~: Overlapping

A B

- If $P(A \cap B) = 0$, then the events are Mutually Exclusive (disjoint). This means that… have Nothing nothing in common
- If $P(A \cap B) \neq 0$, then the events are Overlapping. This means that… A & B share outcomes

A B

Examples:

1. The table below gives the probability of each color for a randomly chosen milk chocolate M&M:

Color:	Brown	Red	Yellow	Green	Orange	Blue
Probability:	0.13	0.13	0.14	0.16	0.20	?

   a) What is the probability of selecting a blue M&M? $P(\text{Blue}) = 1 - 0.76 = 0.24$

   b) What is the probability of selecting a red, yellow, or orange M&M?

   $P(R \cup Y \cup O) = P(R) + P(Y) + P(O) - P(R \cap Y \cap O) = 0.47$

2. The chances Deborah gets promoted is 0.7. The chances Matthew gets promoted is 0.5. The chances they both get promoted is 0.3. What is the probability that at least one of them is promoted?

$P(D) = 0.7 \quad P(M) = 0.5 \quad P(D \cap M) = 0.3$

$P(D \cup M) = P(D) + P(M) - P(D \cap M) =$

$0.7 + 0.5 - 0.3 = \boxed{0.9}$

D M

0.4 0.3 0.2

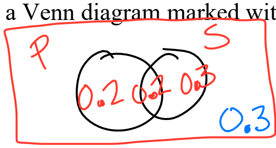
3. Zack has applied to both Princeton and Stanford. He thinks the probability that that Princeton will admit him is 0.4, the probability that Stanford will admit him is 0.5, and the probability that both will admit him is 0.2.

$P(P) = 0.4$

$P(S) = 0.5$

$P(P \cap S) = 0.2$

(a) Make a Venn diagram marked with the given probabilities.



(b) What is the probability that neither university admits Zack?

$P(\text{neither}) = 1 - 0.2 - 0.2 - 0.3 = \boxed{0.3}$

(c) What is the probability that he gets into Stanford but not Princeton?

$P(St) = 0.3$

(d) What is the probability that he gets into Stanford or Princeton?

$P(S \cup P) = P(S) + P(P) - P(S \cap P)$

$0.5 + 0.4 - 0.2 = \boxed{0.7}$

4. 3200 people were surveyed about their ice cream preference. They also had to put their age on the survey. The results are below.

Ice cream preference

Age		Likes Chocolate Ice Cream	Likes Vanilla Ice Cream	Likes Strawberry Ice Cream	Total
	Under 30	604	366	322	1292
	30-51	404	424	286	1114
	Over 51	336	72	386	794
	Total	1344	862	994	3200

a.) What's the probability that a randomly selected individual is over 51?

$P(51+) = 794/3200 = 0.248$

b.) What's the probability that a randomly selected individual likes vanilla ice cream?

$P(\text{Van}) = 862/3200 = \boxed{0.269}$

c.) What's the probability that a randomly selected individual likes chocolate or vanilla ice cream?

$P(C \cup V) = P(C) + P(V) - P(C \cap V) = \frac{1344 + 862}{3200} = \boxed{0.689}$

d.) What's the probability that a randomly selected individual likes strawberry ice cream or is under the age of 30?

$P(S \cup {<30}) = P(S) + P({<30}) - P(S \cap {<30})$ $\boxed{0.614}$

$994 + 1292 - 322 = 1964/3200$

5. Suppose Freddy meets this girl at a party and manages to impress her enough to score her phone number. But when he gets home, he realizes that the last two digits of the phone number were smudged beyond recognition because his hands were ridiculously sweaty. What is the probability that Freddy guesses the last two digits on the first try?

$\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100} = \boxed{0.01}$