UNIT 6 • MODELING GEOMETRY

Lesson 1: Deriving Equations

Common Core Georgia Performance Standards
MCC9–12.G.GPE.1
MCC9–12.G.GPE.2

Essential Questions
1. How can a circle be described in different ways?
2. How can a parabola be described in different ways?
3. How are the definitions of a circle and a parabola similar?
4. How are the definitions of a circle and a parabola different?

WORDS TO KNOW
center of a circle the point in the plane of the circle from which all points on the circle are equidistant. The center is not part of the circle; it is in the interior of the circle.
circle the set of all points in a plane that are equidistant from a reference point in that plane, called the center. The set of points forms a 2-dimensional curve that measures 360°.
directrix of a parabola a line that is perpendicular to the axis of symmetry of a parabola and that is in the same plane as both the parabola and the focus of the parabola; the fixed line referenced in the definition of a parabola
distance formula a formula that states the distance between points \((x_1, y_1)\) and \((x_2, y_2)\) is equal to \(\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}\)
focus of a parabola a fixed point on the interior of a parabola that is not on the directrix of the parabola but is on the same plane as both the parabola and the directrix; the fixed point referenced in the definition of a parabola
general form of an equation of a circle \(Ax^2 + By^2 + Cx + Dy + E = 0\), where \(A = B\), \(A \neq 0\), and \(B \neq 0\)
parabola | the set of all points that are equidistant from a fixed line, called the directrix, and a fixed point not on that line, called the focus. The parabola, directrix, and focus are all in the same plane. The vertex of the parabola is the point on the parabola that is closest to the directrix.

perfect square trinomial | a trinomial of the form \( x^2 + bx + \left( \frac{b}{2} \right)^2 \)

Pythagorean Theorem | a theorem that relates the length of the hypotenuse of a right triangle \( c \) to the lengths of its legs \( a \) and \( b \). The theorem states that \( a^2 + b^2 = c^2 \).

quadratic function | a function that can be written in the form \( f(x) = ax^2 + bx + c \), where \( a \neq 0 \). The graph of any quadratic function is a parabola.

radius | the distance from the center to a point on the circle

standard form of an equation of a circle | \((x - h)^2 + (y - k)^2 = r^2\), where \((h, k)\) is the center and \(r\) is the radius

standard form of an equation of a parabola | \((x - h)^2 = 4p(y - k)\) for parabolas that open up or down; \((y - k)^2 = 4p(x - h)\) for parabolas that open right or left. For all parabolas, \(p \neq 0\) and the vertex is \((h, k)\).

vertex of a parabola | the point on a parabola that is closest to the directrix and lies on the axis of symmetry
Lesson 1: Deriving Equations

Recommended Resources

  
  http://www.walch.com/rr/00071

  This interactive site reviews the standard form of the equation of a circle and how it is derived. It reminds users that the equation is based on the Pythagorean Theorem, and reviews how to expand binomials and simplify the resulting polynomials to obtain the general form, given the standard form. The site also demonstrates how to use the technique of completing the square to obtain the standard form, given the general form. Questions are provided to assess understanding.

  
  http://www.walch.com/rr/00072

  An interactive soccer player kicks a ball that follows a parabolic path. Users can change the values of \( a \), \( b \), and \( c \) to obtain equations of the form \( y = ax^2 + bx + c \), and see how the graph changes to match the equation. This site might be useful as a review of what has already been learned about quadratic functions and their graphs, which are parabolas.

  
  http://www.walch.com/rr/00073

  This site offers a hands-on activity showing how to use the definition of a parabola to draw one. The only required materials are paper, a pencil, and a ruler. The site explains that rays parallel to the axis of symmetry are reflected from the parabola to its focus, and it lists some devices that use that property, such as a satellite dish. Users can assess understanding by answering quiz questions.
Lesson 6.1.1: Deriving the Equation of a Circle

Warm-Up 6.1.1

A video game designer created the following diagram of a target.

1. What is the radius of the circle that contains point $A$? (Hint: Draw a right triangle and use the Pythagorean Theorem.)

2. What is the radius of the circle that contains point $B$?
Lesson 6.1.1: Deriving the Equation of a Circle

Common Core Georgia Performance Standard

MCC9–12.G.GPE.1

Warm-Up 6.1.1 Debrief

A video game designer created the following diagram of a target.

1. What is the radius of the circle that contains point $A$? (*Hint*: Draw a right triangle and use the Pythagorean Theorem.)

The following diagram depicts the triangles for both questions 1 and 2.
Lesson 1: Deriving Equations

Instruction

**Connection to the Lesson**
- Students will use right triangles and the Pythagorean Theorem to derive equations of circles.
- Students will use specific points on circles such as (4, 3) and the general point \((x, y)\).
Introduction

The graph of an equation in $x$ and $y$ is the set of all points $(x, y)$ in a coordinate plane that satisfy the equation. Some equations have graphs with precise geometric descriptions. For example, the graph of the equation $y = 2x + 3$ is the line with a slope of 2, passing through the point $(0, 3)$. This geometric description uses the familiar concepts of line, slope, and point. The equation $y = 2x + 3$ is an algebraic description of the line.

In this lesson, we will investigate how to translate between geometric descriptions and algebraic descriptions of circles. We have already learned how to use the Pythagorean Theorem to find missing dimensions of right triangles. Now, we will see how the Pythagorean Theorem leads us to the distance formula, which leads us to the standard form of the equation of a circle.

Key Concepts

- The standard form of the equation of a circle is based on the distance formula.
- The distance formula, in turn, is based on the Pythagorean Theorem.
- The **Pythagorean Theorem** states that in any right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

\[
a^2 + b^2 = c^2
\]
If \( r \) represents the distance between the origin and any point \((x, y)\), then \( x^2 + y^2 = r^2 \).

- \( x \) can be positive, negative, or zero because it is a coordinate.
- \( y \) can be positive, negative, or zero because it is a coordinate.
- \( r \) cannot be negative because it is a distance.

A **circle** is the set of all points in a plane that are equidistant from a reference point in that plane, called the center. The set of points forms a 2-dimensional curve that measures 360°.

The **center of a circle** is the point in the plane of the circle from which all points on the circle are equidistant. The center is in the interior of the circle.

The **radius** of a circle is the distance from the center to a point on the circle.

For a circle with center \((0, 0)\) and radius \(r\), any point \((x, y)\) is on that circle if and only if \( x^2 + y^2 = r^2 \).

The distance formula is used to find the distance between any two points on a coordinate plane.

The **distance formula** states that the distance \( d \) between \( A (x_1, y_1) \) and \( B (x_2, y_2) \) is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]

The distance formula is based on the Pythagorean Theorem.
Look at this diagram of a right triangle with points $A$, $B$, and $C$. The distance $d$ between points $A$ and $B$ is unknown.

The worked example that follows shows how the distance formula is derived from the Pythagorean Theorem, using the points from the diagram to find $d$:

\[ AB^2 = BC^2 + AC^2 \]  
Pythagorean Theorem

\[ d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2 \]
Substitute values for sides $AB$, $BC$, and $AC$ of the triangle.

\[ d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 \]  
Simplify. All squares are nonnegative.

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
Take the square of each side of the equation to arrive at the distance formula.
For a circle with center \((h, k)\) and radius \(r\), any point \((x, y)\) is on that circle if and only if 
\[ \sqrt{(x-h)^2 + (y-k)^2} = r. \]
Squaring both sides of this equation yields the **standard form of an equation of a circle** with center \((h, k)\) and radius \(r\): 
\[ (x-h)^2 + (y-k)^2 = r^2. \]

If a circle has center \((0, 0)\), then its equation is 
\[ x^2 + y^2 = r^2. \]

If the center and radius of a circle are known, then either of the following two methods can be used to write an equation for the circle:
- Apply the Pythagorean Theorem to derive the equation.
- Or, substitute the center coordinates and radius directly into the standard form.

The **general form of an equation of a circle** is 
\[ Ax^2 + By^2 + Cx + Dy + E = 0, \]
where \(A = B\), \(A \neq 0\), and \(B \neq 0\).

If any one of the following three sets of facts about a circle is known, then the other two can be determined:
- center \((h, k)\) and radius \(r\)
- standard equation: 
  \[ (x-h)^2 + (y-k)^2 = r^2 \]
- general equation: 
  \[ Ax^2 + By^2 + Cx + Dy + E = 0 \]
The general form of the equation of a circle comes from expanding the standard form of the equation of the circle.

The standard form of the equation of a circle comes from completing the square from the general form of the equation of a circle.

Every perfect square trinomial has the form \( x^2 + bx + \left( \frac{b}{2} \right)^2 \) because it is the square of a binomial: \( x^2 + bx + \left( \frac{b}{2} \right)^2 = \left( x + \frac{b}{2} \right)^2 \).

Completing the square is the process of determining the value of \( \left( \frac{b}{2} \right)^2 \) and adding it to \( x^2 + bx \) to form the perfect square trinomial, \( x^2 + bx + \left( \frac{b}{2} \right)^2 \).

Common Errors/Misconceptions

- confusing the radius with the square of the radius
- forgetting to square half the coefficient of \( x \) when completing the square
- neglecting to square the denominator when squaring a fraction
Guided Practice 6.1.1

Example 1

Derive the standard equation of the circle with center $(0, 0)$ and radius 5.

1. Sketch the circle.
2. Use the Pythagorean Theorem to derive the standard equation.

In order to use the Pythagorean Theorem, there must be a right triangle.

To create a right triangle, draw a line from point \((x, y)\) that is perpendicular to the horizontal line through the circle. Label the resulting sides of the triangle \(x\) and \(y\).

Substitute the values for each side of the triangle into the formula for the Pythagorean Theorem, \(a^2 + b^2 + c^2\).

\[
\begin{align*}
a^2 + b^2 + c^2 & \quad \text{Pythagorean Theorem} \\
x^2 + y^2 &= 5^2 & \text{Substitute values from the triangle.} \\
x^2 + y^2 &= 25 & \text{Simplify.}
\end{align*}
\]

The standard equation is \(x^2 + y^2 = 25\).
Example 2

Derive the standard equation of the circle with center (2, 1) and radius 4. Then use a graphing calculator to graph your equation.

1. Sketch the circle.

2. Use the Pythagorean Theorem to derive the standard equation.

Create a right triangle. Draw lines from point \((x, y)\) and point \((2, 1)\) that meet at a common point and are perpendicular to each other.

The length of the base of the triangle is equal to the absolute value of the difference of the \(x\)-coordinates of the endpoints.

The height of the triangle is equal to the absolute value of the difference of the \(y\)-coordinates of the endpoints.

(continued)
Substitute the resulting values for the sides of the triangle into the Pythagorean Theorem.

\[ a^2 + b^2 + c^2 \]  

\[ |x-2|^2 + |y-1|^2 = 4^2 \]  

\[ (x-2)^2 + (y-1)^2 = 4^2 \]  

All squares are nonnegative, so replace the absolute value symbols with parentheses.

\[ (x-2)^2 + (y-1)^2 = 16 \]  

Simplify.

The standard equation is \((x-2)^2 + (y-1)^2 = 16\).
3. Solve the standard equation for $y$ to obtain functions that can be graphed.

\[(x - 2)^2 + (y - 1)^2 = 16\]

**Standard equation**

\[(y - 1)^2 = 16 - (x - 2)^2\]

**Subtract** $(x - 2)^2$ **from both sides.**

\[y - 1 = \pm \sqrt{16 - (x - 2)^2}\]

**If** $a^2 = b^2$, **then** $a = \pm b$.

\[y = 1 \pm \sqrt{16 - (x - 2)^2}\]

**Add 1 to both sides to solve for** $y$.

4. Now graph the two functions, $y = 1 + \sqrt{16 - (x - 2)^2}$ and $y = 1 - \sqrt{16 - (x - 2)^2}$.

**On a TI-83/84:**

- **Step 1:** Press [Y=].
- **Step 2:** At Y₁, type in $[1] + \sqrt{[16][–][(|[X, T, \Theta, n][–][2][)]][x^2]]}$.
- **Step 3:** At Y₂, type in $[1] - \sqrt{[16][–][(|[X, T, \Theta, n][–][2][)]][x^2]]}$.
- **Step 4:** Press [WINDOW] to change the viewing window.
- **Step 5:** At Xmin, enter [–][9].
- **Step 6:** At Xmax, enter [9].
- **Step 7:** At Xscl, enter [1].
- **Step 8:** At Ymin, enter [–][6].
- **Step 9:** At Ymax, enter [6].
- **Step 10:** At Yscl, enter [1].
- **Step 11:** Press [GRAPH].

**On a TI-Nspire:**

- **Step 1:** Press the [home] key.
- **Step 2:** Arrow to the graphing icon and press [enter].
- **Step 3:** At the blinking cursor at the bottom of the screen, type $[1][+] [\sqrt{[16][–][(|[x][–][2][)]][x^2]}][enter]$. 

*(continued)*
Step 4: Move the cursor to the bottom left of the screen and click on the double right-facing arrows.

Step 5: At the blinking cursor, type \([1][-][\sqrt{16}][][-][(|x|][-][2])][x^2][(][][enter].

Step 6: Change the viewing window by pressing [menu], using the arrows to navigate down to number 4: Window/Zoom, and clicking the center button of the navigation pad.

Step 7: Choose 1: Window settings by pressing the center button.

Step 8: Enter in an appropriate XMin value, –9, by pressing [(-)] and [9], then press [tab].

Step 9: Enter in an appropriate XMax value, [9], then press [tab].

Step 10: Leave the XScale set to “Auto.” Press [tab] twice to navigate to YMin and enter an appropriate YMin value, –6, by pressing [(-)] and [6].

Step 11: Press [tab] to navigate to YMax. Enter [6]. Press [tab] twice to leave YScale set to “auto” and to navigate to “OK.”

Step 12: Press [enter].

Step 13: Press [menu] and select 2: View and 5: Show Grid.
Example 3

Write the standard equation and the general equation of the circle that has center \((-1, 3)\) and passes through \((-5, 5)\).

1. Sketch the circle.
2. Use the distance formula to find the radius, \( r \).

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

Distance formula

\[
r = \sqrt{(-5) - (-1)^2 + (5 - 3)^2}
\]

Substitute \((-1, 3)\) and \((-5, 5)\) for \((x_1, y_1)\) and \((x_2, y_2)\).

\[
r = \sqrt{(-4)^2 + (2)^2}
\]

Simplify.

\[
r = \sqrt{16 + 4}
\]

\[
r = \sqrt{20}
\]

Write 20 as a product with a perfect square factor.

\[
r = \sqrt{4 \cdot 5}
\]

Apply the property \( \sqrt{ab} = \sqrt{a} \cdot \sqrt{b} \).

\[
r = \sqrt{4} \cdot \sqrt{5}
\]

Simplify.

\[
r = 2\sqrt{5}
\]

3. Substitute the center and radius directly into the standard form.

\[
(x - h)^2 + (y - k)^2 = r^2
\]

Standard form

\[
\left[ x - (-1) \right]^2 + (y - 3)^2 = (2\sqrt{5})^2
\]

Substitute values into the equation, using the center \((-1, 3)\), and the radius \(2\sqrt{5}\).

\[
(x + 1)^2 + (y - 3)^2 = 20
\]

Simplify to obtain the standard equation.

The standard equation is \((x + 1)^2 + (y - 3)^2 = 20\).
4. Square the binomials and rearrange terms to obtain the general form.

\[(x + 1)^2 + (y - 3)^2 = 20\]  
Standard equation

\[(x + 1)(x + 1) + (y - 3)(y - 3) = 20\]  
Expand the factors.

\[x^2 + 2x + 1 + y^2 - 6y + 9 = 20\]  
Square the binomials to obtain trinomials.

\[x^2 + 2x + y^2 - 6y + 10 = 20\]  
Combine the constant terms on the left side of the equation.

\[x^2 + 2x + y^2 - 6y - 10 = 0\]  
Subtract 20 from both sides to get 0 on the right side.

\[x^2 + y^2 + 2x - 6y - 10 = 0\]  
Rearrange terms in descending order to obtain the general equation.

The general equation is \(x^2 + y^2 + 2x - 6y - 10 = 0\).
Example 4
Find the center and radius of the circle described by the equation \(x^2 + y^2 - 8x + 2y + 2 = 0\).

1. Rewrite the equation in standard form.

\[
\begin{align*}
\text{General form of the equation} & : x^2 + y^2 - 8x + 2y + 2 = 0 \\
\text{Subtract 2 from both sides} & : x^2 + y^2 - 8x + 2y = -2 \\
\text{Group same-variable terms.} & : x^2 - 8x + y^2 + 2y = -2
\end{align*}
\]

Next, complete the square for both variables. Add the same values to both sides of the equation as shown:

\[
\begin{align*}
x^2 - 8x + \left(\frac{-8}{2}\right)^2 + y^2 + 2y + \left(\frac{2}{2}\right)^2 &= -2 + \left(\frac{-8}{2}\right)^2 + \left(\frac{2}{2}\right)^2 \\
x^2 - 8x + 16 + y^2 + 2y + 1 &= -2 + 16 + 1 \\
(x - 4)^2 + (y + 1)^2 &= 15
\end{align*}
\]

The standard equation is \((x - 4)^2 + (y + 1)^2 = 15\).

2. Determine the center and radius.

\[
\begin{align*}
(x - 4)^2 + (y + 1)^2 &= 15 & \text{Write the standard equation from step 1.} \\
(x - 4)^2 + \left[y - \left(-1\right)\right]^2 &= \left(\sqrt{15}\right)^2 & \text{Rewrite to match the form \((x - h)^2 + (y - k)^2 = r^2\).} \\
\end{align*}
\]

For the equation \((x - h)^2 + (y - k)^2 = r^2\), the center is \((h, k)\) and the radius is \(r\), so for the equation \((x - 4)^2 + \left[y - \left(-1\right)\right]^2 = \left(\sqrt{15}\right)^2\), the center is \((4, -1)\) and the radius is \(\sqrt{15}\).
Example 5

Find the center and radius of the circle described by the equation $4x^2 + 4y^2 + 20x - 40y + 116 = 0$.

1. Rewrite the equation in standard form.

   $4x^2 + 4y^2 + 20x - 40y + 116 = 0$  \hspace{1cm} \text{General form of the equation}

   $x^2 + y^2 + 5x - 10y + 29 = 0$ \hspace{1cm} \text{Divide each term on both sides by 4 to make the leading coefficient 1.}

   $x^2 + y^2 + 5x - 10y = -29$ \hspace{1cm} \text{Subtract 29 from both sides to get the constant term on one side.}

   $x^2 + 5x + y^2 - 10y = -29$ \hspace{1cm} \text{Combine like terms.}

Next, complete the square for both variables. Add the same values to both sides of the equation, as shown below:

\[
x^2 + 5x + \left(\frac{5}{2}\right)^2 + y^2 - 10y + \left(\frac{-10}{2}\right)^2 = -29 + \left(\frac{5}{2}\right)^2 + \left(\frac{-10}{2}\right)^2
\]

\[
x^2 + 5x + \frac{25}{4} + y^2 - 10y + 25 = -29 + \frac{25}{4} + 25 \hspace{1cm} \text{Simplify the equation from above.}
\]

\[
\left(x + \frac{5}{2}\right)^2 + (y - 5)^2 = \frac{9}{4}
\]

Write the perfect square trinomials as squares of binomials.

The standard equation is $\left(x + \frac{5}{2}\right)^2 + (y - 5)^2 = \frac{9}{4}$. 
2. Determine the center and radius.

\[
\left( x + \frac{5}{2} \right)^2 + (y - 5)^2 = \frac{9}{4}
\]

Write the standard equation from step 1.

\[
\left[ x - \left( -\frac{5}{2} \right) \right]^2 + (y - 5)^2 = \left( \frac{3}{2} \right)^2
\]

Rewrite to match the form \( (x - h)^2 + (y - k)^2 = r^2 \).

For the equation \((x - h)^2 + (y - k)^2 = r^2\), the center is \((h, k)\) and the radius is \(r\), so for the equation \[
\left[ x - \left( -\frac{5}{2} \right) \right]^2 + (y - 5)^2 = \left( \frac{3}{2} \right)^2,
\] the center is \(-\frac{5}{2}, 5\) and the radius is \(\frac{3}{2}\).
Problem-Based Task 6.1.1: Nurturing an Investment

Anna’s landscaping company has a contract to improve and maintain a municipal park. Anna made a scale drawing of the park on a coordinate system, using meters as the unit of distance. She has already installed two permanent sprinkler outlets. Sprinkler 1 waters inside the region whose boundary has the equation \( x^2 + y^2 - 20x - 20y + 136 = 0 \). Sprinkler 2 waters inside the region whose boundary has the equation \( x^2 + y^2 - 50x - 24y + 669 = 0 \). Anna bought an expensive tree and she wants to plant it at the point (17, 8), where she thinks it will be watered by both sprinklers. Will the tree be watered by both sprinklers at that point? Draw a sketch that illustrates your answer.
Problem-Based Task 6.1.1: Nurturing an Investment

Coaching

a. What geometric figures are represented by the given equations? How do you know?

b. What facts do you need to know in order to determine the regions watered by the sprinklers?

c. What form of equation do you need to find the facts referenced in part b?

d. What are the main steps needed to obtain the form referenced in part c?

e. For sprinkler 1, apply the necessary steps to obtain the form referenced in parts c and d.

f. What is the geometric description of the region watered by sprinkler 1?

g. What is the geometric description of the region watered by sprinkler 2?

h. What is the distance between sprinkler 1 and the planned location of the tree?

i. Will the tree be watered by sprinkler 1? Explain.

j. Will the tree be watered by sprinkler 2? Explain.

k. Will the tree be watered by both sprinklers? Draw a sketch that illustrates your answer.
Problem-Based Task 6.1.1: Nurturing an Investment

Coaching Sample Responses

a. What geometric figures are represented by the given equations? How do you know?

The figures are circles. The equations are in the general form of an equation of a circle.

b. What facts do you need to know in order to determine the regions watered by the sprinklers?

You need to know the center and radius of each circle.

c. What form of equation do you need to find the facts referenced in part b?

You need the standard form of an equation of a circle.

d. What are the main steps needed to obtain the form referenced in part c?

For each given equation, add or subtract as needed from both sides to get the constant term on one side. Group same-variable terms. Complete the square for both variables, making sure to add the same values to both sides of the equation. Write the perfect square trinomials as squares of binomials.

e. For sprinkler 1, apply the necessary steps to obtain the form referenced in parts c and d.

\[ x^2 + y^2 - 20x - 20y + 136 = 0 \]
\[ x^2 + y^2 - 20x - 20y = -136 \]
\[ x^2 - 20x + y^2 - 20y = -136 \]

Next, complete the square for both variables. Add the same values to both sides of the equation, as shown below:

\[ x^2 - 20x + \left(\frac{-20}{2}\right)^2 + y^2 - 20y + \left(\frac{-20}{2}\right)^2 = -136 + \left(\frac{-20}{2}\right)^2 + \left(\frac{-20}{2}\right)^2 \]

\[ x^2 - 20x + 100 + y^2 - 20y + 100 = -136 + 100 + 100 \]
\[ (x - 10)^2 + (y - 10)^2 = 64 \]

The standard equation is \( (x - 10)^2 + (y - 10)^2 = 64 \).
f. What is the geometric description of the region watered by sprinkler 1?

Use the standard equation to determine the center and radius of the region watered by sprinkler 1. Sprinkler 1 waters the interior of the circle with center (10, 10) and radius 8 meters.

g. What is the geometric description of the region watered by sprinkler 2?

Convert the general equation to standard form, and then determine the center and radius of the region watered by sprinkler 2.

\[
\begin{align*}
  x^2 + y^2 - 50x - 24y + 669 &= 0 \\
  x^2 + y^2 - 50x - 24y &= -669 \\
  x^2 - 50x + y^2 - 24y &= -669 \\
\end{align*}
\]

Next, complete the square for both variables. Add the same values to both sides of the equation, as shown:

\[
\begin{align*}
  x^2 - 50x + \left(\frac{-50}{2}\right)^2 + y^2 - 24y + \left(\frac{-24}{2}\right)^2 &= -669 + \left(\frac{-50}{2}\right)^2 + \left(\frac{-24}{2}\right)^2 \\
  x^2 - 50x + 625 + y^2 - 24y + 144 &= -669 + 625 + 144 \\
  (x - 25)^2 + (y - 12)^2 &= 100 \\
\end{align*}
\]

Sprinkler 2 waters the interior of the circle with center (25, 12) and radius 10 meters.

h. What is the distance between sprinkler 1 and the planned location of the tree?

Use the distance formula, substituting points (10, 10) and (17, 8) for \((x_1, y_1)\) and \((x_2, y_2)\).

\[
\begin{align*}
  d &= \sqrt{(17 - 10)^2 + (8 - 10)^2} \\
  d &= \sqrt{49 + 4} \\
  d &= \sqrt{53} \text{ meters} \\
\end{align*}
\]

i. Will the tree be watered by sprinkler 1? Explain.

Yes; the radius of the circular region watered by sprinkler 1 is 8 meters, or \(\sqrt{64}\) meters. The tree will be \(\sqrt{53}\) meters from the sprinkler, which is less than the radius.
j. Will the tree be watered by sprinkler 2? Explain.

Use the distance formula, substituting points (25, 12) and (17, 8) for \((x_1, y_1)\) and \((x_2, y_2)\).

\[
d = \sqrt{(17 - 25)^2 + (8 - 12)^2}
\]

\[
d = \sqrt{64 + 16}
\]

\[
d = \sqrt{80} \text{ meters}
\]

Yes; the radius of the circular region watered by sprinkler 2 is 10 meters, or \(\sqrt{100} \text{ meters}\). The tree will be \(\sqrt{80}\) meters from the sprinkler, which is less than the radius.

k. Will the tree be watered by both sprinklers? Draw a sketch that illustrates your answer.

Yes, the tree will be watered by both sprinklers. Sample sketch:

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**Recommended Closure Activity**

Select one or more of the essential questions for a class discussion or as a journal entry prompt.