GSE Algebra 1

8.9 – Quadratic Formula Proof

Notes

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Use the quadratic formula to solve the following problems.

1) $2x^2 + 39 = -18x$

$+18x \quad +18x$

$2x^2 + 18x + 39 = 0$

$a = 2$, $b = 18$, $c = 39$

$x = \frac{-(18) \pm \sqrt{(18)^2 - 4(2)(39)}}{2(2)}$

$\frac{-18 \pm \sqrt{12}}{4} = \frac{-18 \pm 2\sqrt{3}}{4} \div 2$

$\frac{-9 \pm \sqrt{3}}{2}$

2) $5x^2 + 3x + 1 = 0$

$a = 5$, $b = 3$, $c = 1$

$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(5)(1)}}{2(5)}$

$\frac{-3 \pm \sqrt{-11}}{10}$

No Solution

no real answers

3) $6x^2 + 11x - 2 = 0$

$a = 6$

$b = 11$

$c = -2$

$x = \frac{-(11) \pm \sqrt{(11)^2 - 4(6)(-2)}}{2(6)}$

$\frac{-11 \pm \sqrt{169}}{12} = \frac{-11 \pm 13}{12}$

$\frac{-11+13}{12} = \frac{2}{12} = \frac{1}{6}$

$\frac{-11-13}{12} = \frac{-24}{12} = -2$

4) $16x^2 + 9 = 24x$

$-24x \quad -24x$

$16x^2 - 24x + 9 = 0$

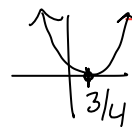
$a = 16$

$b = -24$

$c = 9$

$x = \frac{-(-24) \pm \sqrt{(-24)^2 - 4(16)(9)}}{2(16)}$

$\frac{24 \pm \sqrt{0}}{32} = \frac{24}{32} = \frac{3}{4}$



Now let's try a couple word problems using the quadratic formula.

5) The height of a flare that Mrs. Forrester has fired from the deck of a sinking ship can be modeled by $h = -16t^2 + 104t + 56$, where h is the height of the flare above the water and t is the time in seconds. Find the time it takes the flare to hit the water.

height = 0

7 secs

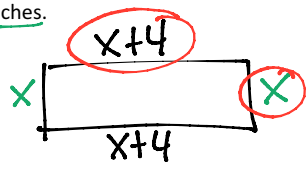
$-16t^2 + 104t + 56 = 0$

$a = -16$, $b = 104$, $c = 56$

$x = \frac{-(104) \pm \sqrt{(104)^2 - 4(-16)(56)}}{2(-16)} = \frac{-104 \pm \sqrt{14400}}{-32} = \frac{-104 \pm 120}{-32}$

$\frac{-104+120}{-32}$ (+) $\quad \frac{-104-120}{-32} = 7$

6) The length of a rectangle exceeds it width by 4 inches. Find the dimensions of the rectangle if its area is 96 square inches.



$x(x+4) = 96$

$x^2 + 4x = 96$

$-96 \quad -96$

$x^2 + 4x - 96 = 0$

$a = 1$, $b = 4$, $c = -96$

Area $= l \cdot w$

$x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-96)}}{2(1)}$

$\frac{-4 \pm \sqrt{400}}{2}$

$\frac{-4 \pm 20}{2}$

$\frac{-4+20}{2} = 8$

$\frac{-4-20}{2} = -12$

$x = 8$ in

$x + 4$

$8 + 4 = 12$ in

8 x 12 in rectangle

$= 96$

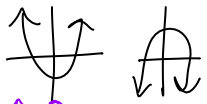
Discrimant

located under $\sqrt{}$

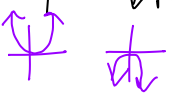
b^2-4ac

3 conditions:

$b^2-4ac > 0 \rightarrow$ 2 real solutions →



$b^2-4ac = 0 \rightarrow$ 1 real solution →



$b^2-4ac < 0$ (neg #) → No real solutions, 2 imaginary solutions

Find Discriminant
Tell what kind of solutions

① $4x^2-4x+9=0$ $\quad b^2-4ac$

$a=4$
$b=-4$
$c=9$

$(-4)^2-4(4)(9) = -128 < 0$ 2 imaginary solutions

② $x^2+4x+4=0$ $\quad b^2-4ac$

$a=1$
$b=4$
$c=4$

$(4)^2-4(1)(4) = 16-16=0$ 1 real solution

③ $9x^2+2x-7=0$ $\quad b^2-4ac$

$a=9$
$b=2$
$c=-7$

$(2)^2-4(9)(-7) = 256$ 2 real solutions

① Start in standard form : $ax^2+bx+c=0$

② Complete the square

③ End : quadratic formula → $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Start

$ax^2+bx+c=0$

$-c \quad -c$

$\frac{ax^2}{a}+\frac{bx}{a}=-c$

$a\left(x^2+\frac{b}{a}x+\frac{b^2}{4a^2}\right)=-c+a\left(\frac{b^2}{4a^2}\right)$

$x \cdot x = x^2$; $\frac{b}{2a}+\frac{b}{2a}=\frac{b}{a}$; $\frac{b}{a}\div\frac{2}{1}$

KCF → $\frac{b}{a}\cdot\frac{1}{2}=\left(\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}$

$a\left(x+\frac{b}{2a}\right)\left(x+\frac{b}{2a}\right)=-c+\frac{a}{1}\cdot\frac{b^2}{4a^2}=\frac{ab^2}{4a^2}=\frac{bb}{4a}=\frac{b^2}{4a}$

$\frac{a\left(x+\frac{b}{2a}\right)^2}{a}=\frac{-c}{a}+\frac{\frac{b^2}{4a}}{a}$

KCF $\frac{b^2}{4a}\cdot\frac{1}{a}=\frac{b^2}{4a^2}$

$\left(x+\frac{b}{2a}\right)^2=\frac{-c}{a}+\frac{b^2}{4a^2}$

$\frac{4a}{4a}\cdot\frac{-c}{a}+\frac{b^2}{4a^2}$

$\frac{-c4a}{4a^2}+\frac{b^2}{4a^2}$

$\frac{b^2-4ac}{4a^2}$

$\sqrt{\left(x+\frac{b}{2a}\right)^2}=\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$

$x+\frac{b}{2a}=\frac{\sqrt{b^2-4ac}}{2a}$

$-\frac{b}{2a} \quad -\frac{b}{2a}$

$x=-\frac{b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

Quadratic Formula