Section 5.3 Notes: General Probability Rules

In this section we will consider some additional laws that govern assignment of probabilities. The purpose of learning more laws of probability is to be able to give probability models for more complex random phenomena.

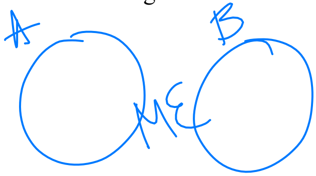
Recall the Addition Rule:

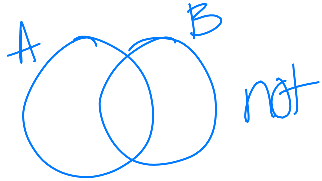
If two events have NO outcomes in common, the probability that one **OR** the other occurs is the SUM of their individual probabilities.

In symbols: If A and B are "disjoint" a.k.a. "mutually exclusive" P(A or B) = P(A) + P(B)

Note: This rule is also known as the addition rule for **mutually exclusive** events**.**

Draw two Venn diagrams. One illustrating disjoint events and one that illustrates events that are not disjoint.





Now think about the addition rule for non-disjoint (overlapping) events. Do you think the same rule still applies?

$P(A \text{ OR } B) = P(A) + P(B) - \boxed{P(A \cap B)}$

This leads us to the **General Addition Rule for Two Events:**

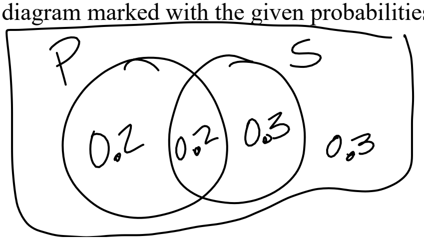
$P(A \text{ OR } B) = P(A) + P(B)$ ← ME/disjoint

$P(A \text{ OR } B) = P(A) + P(B) - P(A \cap B)$ ← overlap

Examples:

1.) Zack has applied to both Princeton and Stanford. He thinks the probability that that Princeton will admit him is 0.4, the probability that Stanford will admit him is 0.5, and the probability that both will admit him is 0.2.

(a) Make a Venn diagram marked with the given probabilities.



(b) What is the probability that neither university admits Zack?

P(Neither) = 0.3

(c) What is the probability that he gets into Stanford but not Princeton?

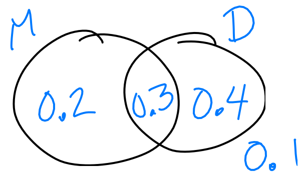
$P(S \cap P^c) = 0.3$

2.) Deborah and Matthew are anxiously awaiting word on whether they have been made partners in their law firm. Deborah guesses that her probability of making partner is about .7 and Matthew's is about .5. She also guesses that the probability of them both making partner is .3.

(a) What is the probability that at least one of the employees is promoted?

P(at least 1) = 1 − P(none) 1 − 0.1 = $\boxed{0.9}$

(b) Make a Venn diagram marked with the given probabilities.



Conditional Probability

Sometimes the knowledge that one event has occurred changes the probability that another event will occur.

- The probability of being in a car accident increases if you know that it is raining outside
- Suppose that the pass rate on the AP is exam is 80%. That is, for a randomly selected student, P(pass) = .80. However, if you know that the student got a B in the class, then the probability increases 95%. That is, for a randomly selected student, P(pass given that you have a B) = .95.
  - Notation: P(pass | B) = .95

The probability we assign to an event can change if we know that some other even has occurred. This idea is the key to many applications of probability. This is called Conditional Probability

Notation: P(A|B) OR P(B|A)

Think back to the beginning of this chapter when we calculated percentages using two-way tables. Technically, you were calculating conditional probabilities.

Example:
Refer to the table to answer the following questions. 3200 people were surveyed.

Ice cream preference

Age		Likes Chocolate Ice Cream	Likes Vanilla Ice Cream	Likes Strawberry Ice Cream	Total
	Under 30	**604**	**366**	**322**	**1292**
	30-51	**404**	**424**	**286**	**1114**
	Over 51	**336**	**72**	**386**	**794**
	Total	**1344**	**862**	**994**	**3200**

a.) Find the probability that a randomly selected individual is over 51. 0.248

b.) Find the probability that a randomly selected individual likes vanilla ice cream. 0.269

c.) Find the probability that a randomly selected individual is under 30 or likes chocolate ice cream. 0.635 1292 + 1344 − 604

d.) What is the probability you choose someone who likes strawberry ice cream given that you selected someone under 30? 0.249 $\frac{322}{1292}$

e.) What is the probability you selected someone under 30 given that you selected someone who likes strawberry ice cream? 0.324 $\frac{322}{994}$

f.) Suppose you randomly choose an individual who is between 30 and 51, what is the probability the person you chose like chocolate ice cream? 0.363 404/1114

Example:
The following data is about the 2201 passengers on the Titanic. 367 out of 1731 males survived and overall 711 survived (males and females). Express this in two-way table with gender and survival as the variables.

	Alive	Dead	Total
Male	367	1364	1731
Female	344	126	470
total	711	1490	2201

Note: The distributions of the totals for gender and survival are called the MARGINAL distributions. The distributions within each gender or survival category are called CONDITIONAL distributions.

Suppose you randomly selected a name from the Titanic's passenger list.

a. P(survived) = $711/2201 = 32.3\%$

b. P(male) = 78.6%

c. P(female and survived) = $344/2201 = 0.156$

d. P(survived | female) = $\frac{344}{470} = 73\%$

e. P(male | survived) = $\frac{367}{711} = 0.516$

f. P(died | female) = $126/470 = 26.8\%$

g. P(female | died) = $126/1490 = 0.085$

h. P(female | survived) = $\frac{344}{711} = 48.4\%$

The **formula** for Conditional Probability:

Let A and B be two events. The *conditional probability* of event B given that event A has occurred is:

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

So now, using the formula, calculate P(female | survived) from the previous example.

$$\frac{P(F \cap S)}{P(S)} = \frac{344}{711} = 0.484$$

It turns out Conditional Probability can be related to the Multiplication Rule.

Recall the Multiplication Rule:

Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs. If A and B are independent then the probability of A and B is the product of their individual probabilities.

In symbols: If A and B are independent, $\boxed{P(A \text{ and } B) = P(A) \bullet P(B)}$

Note: This rule is known as the multiplication rule for **independent** events.

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

So what happens if the events are not independent ($P(A \cap B) \neq P(A) \cdot P(B)$)

$P(A \cap B) = P(A|B) \cdot P(B)$

This leads us to the **General Multiplication Rule for Any Two Events**

The joint probability that events A and B both happen can be found by

$P(B) = \dfrac{P(A \cap B)}{P(A|B)}$

Equivalently

$$P(A \cap B) = P(A) \bullet P(B \mid A)$$

Here $P(B \mid A)$ is the conditional probability that B occurs, given the information that A occurs.

When do I use which formula?

If events A and B are Independent $P(A \text{ and } B) = P(A) \bullet P(B)$

If events A and B are Conditional $P(A \text{ and } B) = P(A) \bullet P(B \mid A)$

Given

Examples

1.) 29% of Internet users download music files, and 67% of downloaders say they don't care if the music is copyrighted. Find the percent of internet users who download music and don't care about copyright.

$P(\text{Down}) = 0.29$

$P(\text{NC} \mid \text{Down}) = 0.67$

$P(\text{Down} \cap \text{NC})$

$P(\text{NC} \mid \text{Down}) = \dfrac{P(\text{Down} \cap \text{NC})}{P(\text{Down})}$

$0.67 = \dfrac{x}{0.29}$

$\boxed{x = 0.194 \quad 19.4\%}$

3.) Matt McGillicuddy is driving on the highway when he notices 2 gas stations up the road. The probability that the first gas station is open is 0.57. The probability of the second being open is 0.42. What is the probability that both gas stations are closed? (assume independence)

$P(1^{st} \text{ open}) = 0.57$ $\quad P(2^{nd} \text{ open}) = 0.42$

$P(\text{Both closed})$

$1 - (0.57 \cdot 0.42) = \boxed{0.7606}$

Some More Facts about Independence and Dependence: $P(A) \cdot P(B) = P(A \cap B)$

- Two events A and B are **independent** if one event does not affect other
  - In other words, knowing that B occurred doesn't change the probability of A occurring.
  - If you can show the above equation holds true, then you can prove two events are independent.

- Two events A and B are **dependent** if $P(A) \cdot P(B) \neq P(A \cap B)$

  one event does affect the other

Recall this example:

Ice cream preference

Age		Likes Chocolate Ice Cream	Likes Vanilla Ice Cream	Likes Strawberry Ice Cream	Total
	Under 30	604	366	322	1292
	30-51	404	424	286	1114
	Over 51	336	72	386	794
	Total	1344	862	994	3200

Are like chocolate ice cream and being over 51 independent? Justify your answer.

$P(>51) = \frac{794}{3200}$ $P(C) = \frac{1344}{3200}$ $\frac{794 \cdot 1344}{3200 \cdot 3200} = \frac{336}{3200}$

$P(C \cap >51) = \frac{336}{3200}$ $0.104 = 0.105$

$\boxed{\text{Not Independent}}$

Look back at the Titanic Problem. Were gender and survival independent on the Titanic?

$P(Male) = \frac{1731}{2201}$ $P(F) = \frac{470}{2201}$ $P(Surviving) = \frac{711}{2201}$

$P(M \cap S) = \frac{367}{2201}$ $P(F \cap S) = \frac{344}{2201}$

$P(M) \cdot P(S) = P(M \cap S)$

$\frac{1731}{2201} \cdot \frac{711}{2201} = \frac{367}{2201}$

$0.254 = 0.1667$

Not Ind

$P(F) \cdot P(S) = P(F \cap S)$

$\frac{470}{2201} \cdot \frac{711}{2201} = \frac{344}{2201}$

$0.069 = 0.156$

Not Ind

Another Example: Tree Diagrams

Teenagers are the biggest users of online chat rooms. If we look at the age of internet users, 47% of the 18 to 29 age like to chat, as do 21% of those aged 30 to 49 and just 7% of those 50 and over. We also know the percentage of internet users in each age group. 29% of Internet users are aged 18 to 29 (event A_1), another 47% are 30 to 49 (event A_2), and the remaining 24% are 50 and over (event A_3). Let C be the event of a user participating in a chat room.

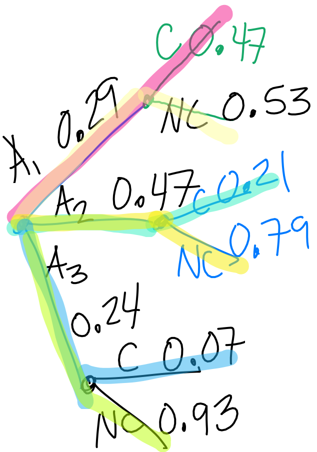
C=Chat Room

First let's determine all the different probabilities the problem has given us so far:

$P(C|A_1)=0.47$ $\quad P(C|A_2)=0.21$ $\quad P(C|A_3)=0.07$

$P(A_1)=0.29$ $\quad P(A_2)=0.47$ $\quad P(A_3)=0.24$

Now let's make a tree diagram of this situation:



$P(A_1\cap C)=0.29\cdot 0.47$

0.1363

$P(A_1\cap C^c)=0.1537$

$P(A_2\cap C)=0.0987$

$P(A_2\cap C^c)=0.3713$

$P(A_3\cap C)=0.0168$

$P(A_3\cap C^c)=0.2232$

What is the probability that a randomly chose user of the Internet participates in chat rooms?

$$P(C)=P(A_1\cap C)+P(A_2\cap C)+P(A_3\cap C)$$

$$=0.2518$$