GSE Algebra 1

8.7 – C.T.S to Vertex form

Name: _____

Complete the square to solve for x. Warmup

$x^2 - 14x - 12 = 0$	$3x^2 - 12x + 1 = 0$	$x^2 + x - 10 = 0$

Now we are going to complete the square and leave it in vertex form.

** **Vertex form**: $y = a(x - h)^2 + k$

Standard → Vertex

$y = x^2 + 10x + 15$

$x^2 + 10x + 15 = 0$

$-15 \quad -15$

$x^2 + 10x + (5)^2 = -15 + 25$

$\frac{10}{2}$

$(5)^2 = 25$

$(x+5)^2 = 10$

$-10 \quad -10$

$(x+5)^2 - 10 = 0$

Vertex: $(-5, -10)$ Max or Min: Min

$y = x^2 - 8x - 4$

$x^2 - 8x - 4 = 0$

$x^2 - 8x + (-4)^2 = 4 + 16$

$\frac{-8}{2}$

$(-4)^2 = 16$

$(x-4)^2 = 20$

$-20 \quad -20$

$(x-4)^2 - 20 = 0$

Vertex: $(4, -20)$ Max or Min: Min

$y = 2x^2 + 12x + 17$

$2x^2 + 12x + 17 = 0$

$-17 \quad -17$

$\frac{2x^2}{2} + \frac{12x}{2} = -17$

$2(x^2 + \frac{6x}{2} + (3)^2) = -17 + 2(9)$

$(3)^2 = 9$

$2(x+3)^2 = 1$

$2(x+3)^2 - 1 = 0$

Vertex: $(-3, -1)$ Min or Max: Min

$y = \frac{1}{2}x^2 - 6x + 11$

$\frac{1}{2}x^2 - 6x + 11 = 0$

$-11 \quad -11$

$\frac{\frac{1}{2}x^2}{\frac{1}{2}} - \frac{6x}{\frac{1}{2}} = -11$

$\frac{1}{2}(x^2 - \frac{12x}{2} + (6)^2) = -11 + \frac{1}{2}(36)$

$(6)^2 = 36$

$\frac{1}{2}(x+6)^2 = 7$

$-7 \quad -7$

$\frac{1}{2}(x+6)^2 - 7 = 0$

Vertex: $(-6, -7)$ Min or Max: Min

$y = -4x^2 - 64x - 156$

$\frac{-4x^2}{-4} - \frac{64x}{-4} = 156$

$-4(x^2 + \frac{16x}{2} + (8)^2) = 156 + -4(64)$

$(8)^2 = 64$

$-4(x+8)^2 = -100$

$+100 \quad +100$

$-4(x+8)^2 + 100 = 0$

Vertex: $(-8, 100)$ Min or max: Max

$y = -x^2 + 4x - 10$

$-x^2 + 4x - 10 = 0$

$+10 \quad +10$

$\frac{-x^2}{-1} + \frac{4x}{-1} = 10$

$-1(x^2 - \frac{4x}{2} + (-2)^2) = 10 + -1(4)$

$(-2)^2 = 4$

$-1(x-2)^2 = 6$

$-6 \quad -6$

$-1(x-2)^2 - 6 = 0$

Vertex: $(2, -6)$ Min or Max: Max

$x^2 - 12x + 32 = 0$

$x^2 - 12x + (-6)^2 = -32 + 36$

$\frac{-12}{2}$

$(-6)^2 = 36$

$(x-6)^2 = 4$

$(x-6)^2 - 4 = 0$

Vertex: $(6, -4)$ Min or Max: Min

$y = -2x^2 - 12x + 8$

$-2x^2 - 12x + 8 = 0$

$-8 \quad -8$

$\frac{-2x^2}{-2} - \frac{12x}{-2} = \frac{-8}{-2}$

$-2(x^2 + 6x + (3)^2) = -8 + -2(9)$

$\frac{6}{2}$

$(3)^2 = 9$

$-2(x+3)^2 = -26$

$+26 \quad +26$

$-2(x+3)^2 + 26 = 0$

Vertex: $(-3, 26)$ Min or Max: Max

Let's look at these **special cases** for complete the square.

$x^2 + 4x + 10 = 0$

$-10 \quad -10$

$x^2 + 4x + (2)^2 = -10 + 4$

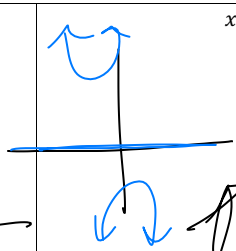
$\frac{4}{2}$

$(2)^2 = 4$

$\sqrt{(x+2)^2} = \sqrt{-6}$

*Cannot take √ of negative number

No Solution



$x^2 - 10x + 65 = 0$

$-65 \quad -65$

$x^2 - 10x + (-5)^2 = -65 + 25$

$\frac{-10}{2}$

$(-5)^2 = 25$

$\sqrt{(x-5)^2} = \sqrt{-40}$

No Solution

$-3x^2 + 12x = 21$

$\frac{-3x^2}{-3} + \frac{12x}{-3} = \frac{21}{-3}$

$-3(x^2 - 4x + (-2)^2) = 21 + -3(4)$

$\frac{-4}{2}$

$(-2)^2 = 4$

$-3x^2 + 12x - 21 = 0$

$+21 \quad +21$

① $\frac{-3(x-2)^2}{-3} = \frac{9}{-3}$

① $\sqrt{(x-2)^2} = \sqrt{-3}$

① No Solution