GSE Algebra 1 **6.5 Notes** Name: _____

You are a tortoise, I am a hare and we are racing. Who wins?

Well that all depends. Let's take a deeper look at that. You say you will win because you are slow and steady but I say I am going to win because I have the speed. The distance from the starting line of the hare is given by the function: $d = t^2$ **(d in meters and t in seconds)**

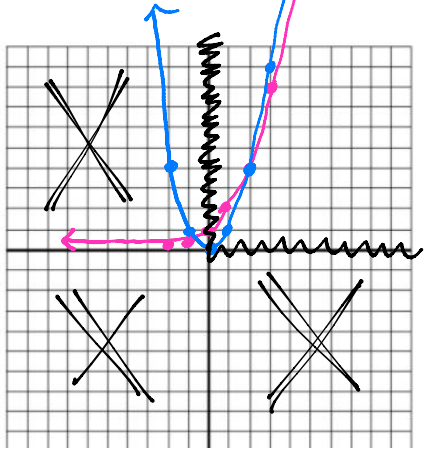
Because I am so confident, being a hare and all, I give you a 1 meter head start. The distance from the starting line of the tortoise (you) including the head start is given by the function:

$d = 2^t$ **(d in meters and t in seconds)**

1) At what times does the tortoise (you) catch up to the hare (me)?
Graph both equations below. Start both equations at x = -2.

Tortoise

x	y
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8
4	16
5	32



Hare

x	y	
-2	4	$(-2)^2$
-1	1	$(-1)^2$
0	0	$(0)^2$
1	1	$(1)^2$
2	4	$(2)^2$
3	9	$(3)^2$
4	16	$(4)^2$
5	25	$(5)^2$

Think about time here. Can we have negative time? Where should we focus our eyes on this graph? Highlight the two graphs in two different colors from where we will inspect them further.

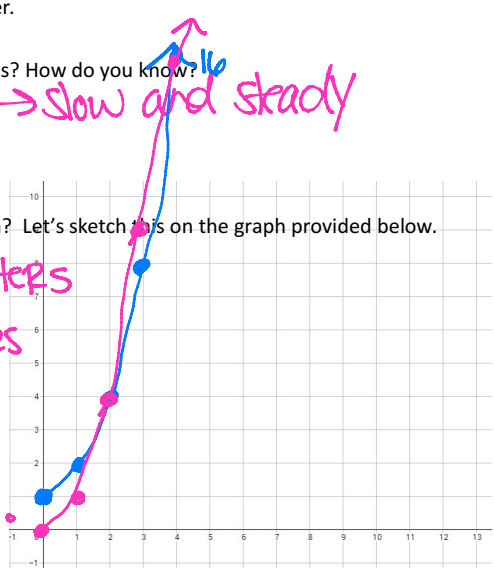
2) If the race course is super long (like miles and miles), who wins? How do you know?

Tortoise → multiplying → slow and steady then takes off

3) At what times are we tied? How can you tell this on the graph? Let's sketch this on the graph provided below.

(2,4) at 2 secs, 4 meters

(4,16) at 4 secs, 16 meters



4) If the racecourse is 15 meters long, who wins you or me? Why?

Hare, at that time, the Hare is moving faster.

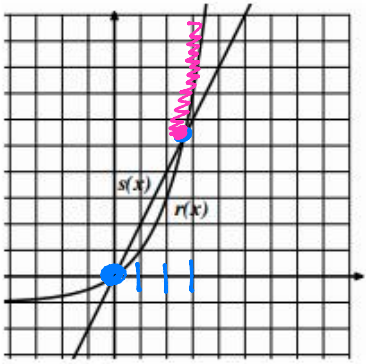
5) Let's look at certain speeds we reach. Fill in the table below. We are going to be finding the rate of change (slope) for each interval. That way we can compare who is faster when ☺

Interval	Tortoise: $d = 2^t$	Hare: $d = t^2$
[0, 2) $\frac{y_2-y_1}{x_2-x_1}$	(0, 1) (2, 4) $\frac{4-1}{2-0}=1.5$	(0, 0) (2, 4) $\frac{4-0}{2-0}=2$ Hare Faster
[2, 4)	(2, 4) (4, 16) $\frac{16-4}{4-2}=6$ tied	(2, 4) (4, 16) $\frac{16-4}{4-2}=6$
[4, oo)	(4, 16) (10, 1024) $\frac{1024-16}{10-4}$ 168 Faster	(4, 16) (10, 100) $\frac{100-16}{10-4}$ 14

Rate of change is another phrase for slope.

How do you find slope? $\frac{y_2-y_1}{x_2-x_1}$

Check out these graphs. Answer the questions that go with them.



Looking at the 2 graphs, which one has a higher rate of change from x = 0 to x = 2.5?

s(x) b/c it's on top, steeper, higher

Which graph has the higher rate of change from x = 2.5 to ∞?

How do you know? r(x) b/c steeper, on top, moving faster

Compare the rates of change for the following money scenarios.
You have $1 when time begins and then meet 3 very nice genies, but you can only accept one of their offers.

Quad will square the amount of days that have gone by, then multiply that by 100, and then give you that amount of money.

Quadratic

$y = 100x^2 + 1$

t	Q(t)
0	1
1	101
2	401
5	2501
9	8101
10	10001
14	19601
15	22501
30	90001

Epo will double your money from the previous day everyday.

Exponential

$y = 1(2)^t$

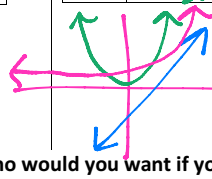
t	E(t)
0	1
1	2
2	4
5	32
9	512
10	1024
14	16384
15	32768
30	1,073,741,824

Luigi will give you $1000 per day.

Linear

$y = 1000x + 1$

t	L(t)
0	1
1	1001
2	2001
5	5001
9	9001
10	10001
14	14001
15	15001
30	30001



exponential has highest Rate of Change → meets Buzz @ ∞

Who would you accept if you only had 10 days to live?

Luigi – has more money on day 9

Who would you want if you wanted fast cash?

Luigi → more money 1st

Who would you accept if you wanted to make the most money in 1 year?

Epo → rolling deep in yo pockets → lots of money @ the end

What is the average rate of change for each function over the given interval?

slope

(3,6) (4,30)

x	f(x)
3	15
4	18
5	21
6	24

+3, +3, +3 → Linear

a) $f(x)$ from 3 to 4 +3

b) $f(x)$ from 5 to 6 +3

x	g(x)
3	2
4	15
5	30
6	47

+13, +15, +17 → +2 → Quad

c) $g(x)$ from 3 to 4 13

d) $g(x)$ from 5 to 6 17

x	h(x)
3	6
4	30
5	150
6	750

$\frac{30-6}{4-3} = 24$

×5, ×5, ×5 → exponential

e) $g(x)$ from 3 to 4 24

f) $g(x)$ from 5 to 6

$\frac{750-150}{6-5} = 600$

$1(2)^{90} =$

1.238×10^{27}

8 1,238,000,000,000,000,000,000,000,000