

The mathematics of chance is called Probability

Activity

As a special promotion for its 20-ounce bottles of soda, a soft drink company printed a message on the inside of each bottle cap. Some of the caps said “please try again” while others said “you’re a winner!” The company advertised the promotion with the slogan “1 in 6 wins.” Seven friends each buy 1 20 ounce soda at a local convenience store. The store clerk is surprised when 3 of them win a prize. Is this group of friends just lucky or is the company’s 1-in-6 claim not correct? Let’s figure that out.

Let’s assume that the company is telling the truth and that every 20 ounce bottle of soda it fills has a 1 in 6 chance of getting a cap that says “you’re a winner!” We can model the status of an individual bottle with a six-sided die; let 1 through 5 represent “please try again” and 6 represent “you’re a winner!”

- 1) Roll your die seven times to imitate the process of the seven friends buying their sodas. How many of them won a prize?
- 2) Repeat step 1 four more times. In your 5 repetitions of this simulation, how many times did three or more of the group win a prize?

- 3) Combine results with the whole class. What percent of the time did the friends come away with three or more prizes, just by chance?

$$\frac{2}{25} = 0.08 = 8\%$$

- 4) Based on your answer in step 3, does it seem plausible that the company is telling the truth, but that the seven friends just got lucky? Let’s describe this all together, combining all our ideas.

$$\frac{1}{6} = 17\% \quad \text{Lucky — Company wants to make \$}$$

Not false advertising

It could also be possible

w/ the people trying to cheat

Based on our class results, we got 8%, which is less than $\frac{1}{6}$ shot of winning, so to get $\frac{3}{7}$ is pretty unlikely.

Chance is one way to avoid bias

Chance behavior is irregular/skewed in the short run but has a regular and normal-ish pattern in the long run.

Activity

Grab a coin. Be sure it has a heads and a tails on it...otherwise how will this work?

What is the chance of getting a heads when you toss a coin? Tails? Are they the same?
 $\frac{1}{2}$ $\frac{1}{2}$ yes

Toss your coin 10 times. Record the number of heads and tails you get in the chart to the right.

What is the proportion of heads? $\frac{4}{10}$ $\frac{5}{10}$ $\frac{4}{10}$ $\frac{3}{10}$ $\frac{6}{10}$

Repeat the process 4 more times (so 40 more throws).

Toss	Outcome
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Toss	11	12	13	14	15	16	17	18	19	20
Outcome										
Toss	21	22	23	24	25	26	27	28	29	30
Outcome										
Toss	31	32	33	34	35	36	37	38	39	40
Outcome										
Toss	41	42	43	44	45	46	47	48	49	50
Outcome										

What do you notice?

Hard to flip coin, over time, # heads + # tails balance out

What would happen if you tossed the coin 100 more times? Does the proportion of heads exactly equal to 0.5? Close to 0.5?

Prop of Heads would be close to 0.5

What does the following statement mean: "If you toss a fair coin, the probability of heads is 0.5"

If you toss a regular coin, it has a 50/50 chance of landing on heads.

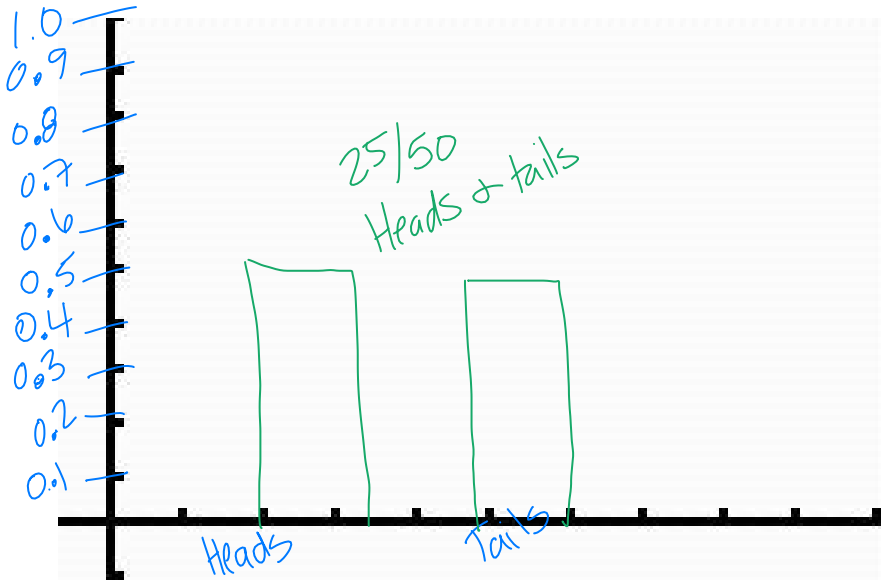
Predict what will happen if you change the probability of heads to 0.3. Now watch the applet to test our predictions.

Tails becomes 0.7, which means you get more tails. Over time → heads should be close to 0.3

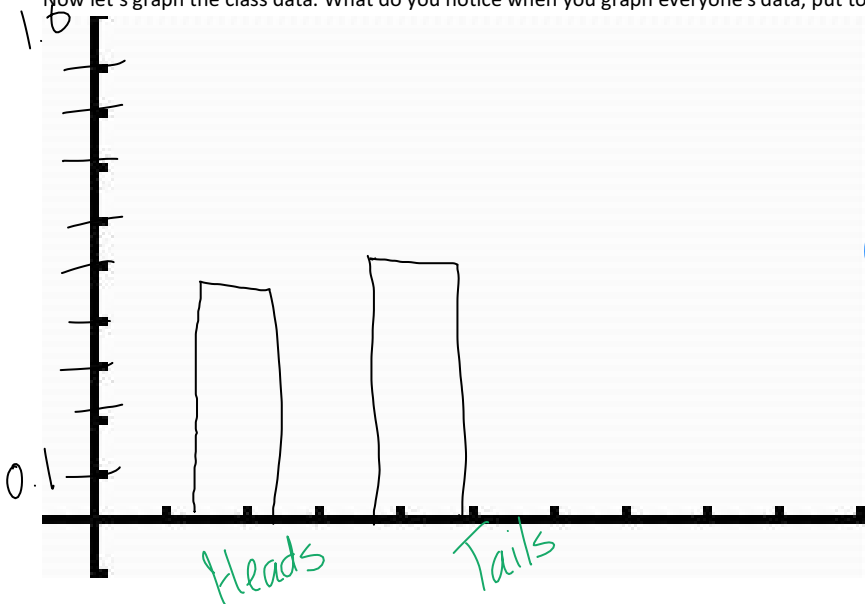
If you toss a coin, you can get heads or tails. If you "toss" a thumbtack, it can land with the point sticking up or the point sticking down. Does that mean that the probability of a toss thumbtack landing point up is 0.5? How could you find out?

Base of thumbtack is heavier, unaccounted variables that effect the throw

Graph your results of your 50 trials below by proportions. Create a bar graph for heads and tails. Remember, bar graphs do not touch.



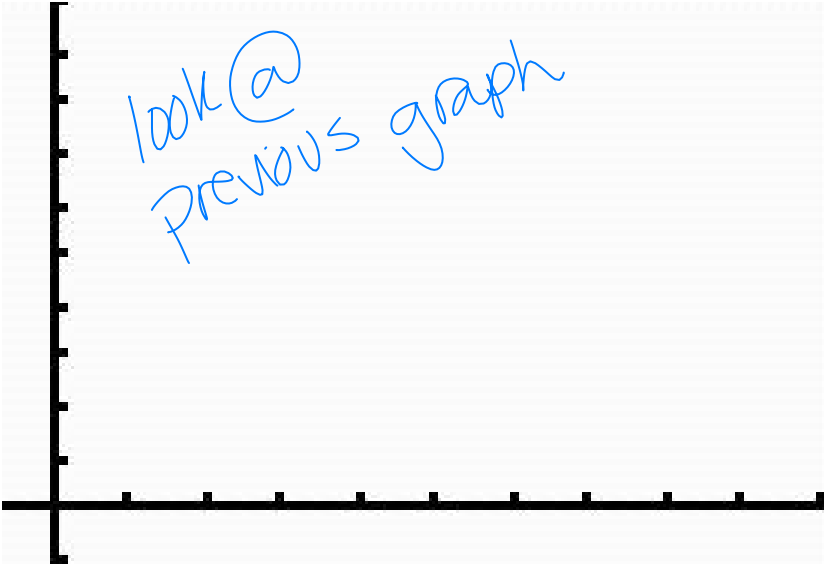
Now let's graph the class data. What do you notice when you graph everyone's data, put together?



H	T
22	28
25	25
20	30
24	26
27	23
<hr/>	<hr/>
250	250
118/250	132/250
0.472	0.528

Cumulative Relative Frequency graphs

Now let's make a cumulative relative frequency chart for the class data.



What do you notice as you are looking at this graph?

← Over time, the more Flips, the closer to 0.5 we get

Law of Large numbers

The fact that the proportion of heads in many tosses eventually closes in at 0.5 is guaranteed because of the law of large numbers. It says that if we observe more and more repetitions of any chance process, the proportion of times that a specific outcome occurs approaches a single value. This value is called **probability**

Define probability below (from the book)

a # b / a that describes the proportion of times the outcome would occur in a long series of repeats.

Outcomes that never occur have probability 0. Outcomes that happen on every repetition have a probability of 1.

Outcomes that happen half of the time have a probability of 0.5