GSE Algebra 1 **6.5 Notes** Name: _____

You are a tortoise, I am a hare and we are racing. Who wins?

Well that all depends. Let's take a deeper look at that. You say you will win because you are slow and steady but I say I am going to win because I have the speed. The distance from the starting line of the hare is given by the function: $\boldsymbol{d = t^2}$ **(d in meters and t in seconds)**

$d=t^2$

Because I am so confident, being a hare and all, I give you a 1 meter head start. The distance from the starting line of the tortoise (you) including the head start is given by the function:

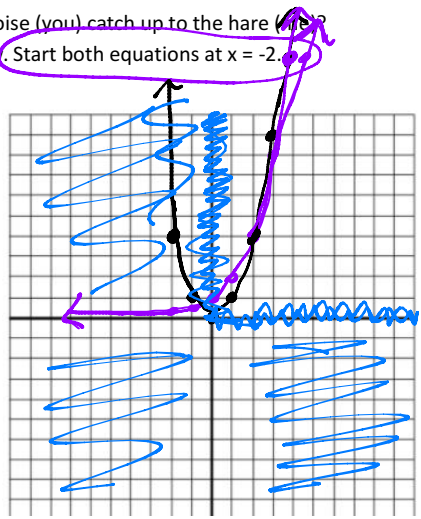
$\boldsymbol{d = 2^t}$ **(d in meters and t in seconds)**

1) At what times does the tortoise (you) catch up to the hare (me)? Graph both equations below. Start both equations at x = -2.

$d=2^t$

Tortoise

x	y
-2	0.25
-1	0.5
0	1
1	2
2	4
3	8
4	16
5	32



$-(2)^2$ $-2\cdot-2=4$

$d=t^2$

Hare

x	y
-2	4
-1	1
0	0
1	1
2	4
3	9
4	16
5	25

$(-2)^2$
$(-1)^2$

Think about time here. Can we have negative time? Where should we focus our eyes on this graph? Highlight the two graphs in two different colors from where we will inspect them further.

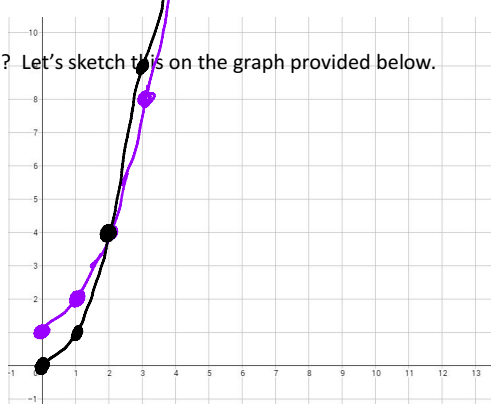
2) If the race course is super long (like miles and miles), who wins? How do you know?

Tortoise → multiply → grows faster

3) At what times are we tied? How can you tell this on the graph? Let's sketch this on the graph provided below.

at 2 secs (2, 4)

at 4 secs (4, 16)



4) If the racecourse is 15 meters long, who wins you or me? Why?

15 meters → Hare

We win → you lose

5) Let's look at certain speeds we reach. Fill in the table below. We are going to be finding the rate of change (slope) for each interval. That way we can compare who is faster when ☺

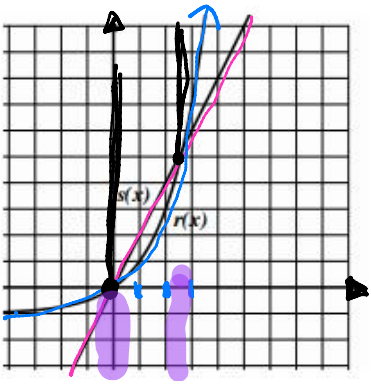
Interval	Tortoise: $d = 2^t$	Hare: $d = t^2$
[0, 2)	$\frac{y_2 - y_1}{x_2 - x_1}$ (0,1) (2,4) $\frac{4-1}{2-0} = 1.5$	(0,0) (2,4) $\frac{4-0}{2-0} = 2$ Hare Faster
[2, 4]	(2,4) (4,16) $\frac{16-4}{4-2}$ 6 Tied	(2,4) (4,16) $\frac{16-4}{4-2}$ 6
[4, oo)	(4,16) (10,1024) $\frac{1024-16}{10-4}$ 168	

Rate of change is another phrase for _____.

How do you find slope?

Check out these graphs. Answer the questions that go with them.

r(x) s(x) → linear



Looking at the 2 graphs, which one has a higher rate of change from x = 0 to x = 2.5?

s(x) is above r(x), so it has a greater rate of change

AKA "slope"

Which graph has the higher rate of change from x = 2.5 to ∞?

How do you know? r(x) exponential function, because the graph is greater than s(x)

Compare the rates of change for the following money scenarios.
You have $1 when time begins and then meet 3 very nice genies, but you can only accept one of their offers.

Quad will square the amount of days that have gone by, then multiply that by 100, and then give you that amount of money.

x = days

$y = 100x^2 + 1$

Quadratic

t	Q(t)
0	1
1	101
2	401
5	2501
9	8101
10	10001
14	19601
15	22501
30	90001

Geo

Exponential

Epo will double your money from the previous day, everyday.

$y = 1(2)^t$

t	E(t)
0	1
1	2
2	4
5	32
9	512
10	1024
14	16384
15	32768
30	$1,073,741,824

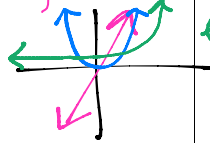
Luigi will give you $1000 per day.

Linear

$y = 1000x + 1$

$1000(5) + 1$

t	L(t)
0	1
1	1001
2	2001
5	5001
9	9001
10	10001
14	14001
15	15001
30	30001



exponential will win when going to ∞

*Multiplying

Who would you accept if you only had 10 days to live?

Luigi or Quad → Luigi has more $ before

Who would you want if you wanted fast cash?

Luigi → more money early

Who would you accept if you wanted to make the most money in 1 year?

Epo → over the long haul, exponential wins

What is the <u>average rate of change</u> for each function over the given interval?

Slope

x	f(x)
3	15
4	18
5	21
6	24

+1, +1, +1 → +3, +3, +3

Linear

same / constant Rate of change

a) $f(x)$ from 3 to 4 +3

b) $f(x)$ from 5 to 6 +3

x	g(x)
3	2
4	15
5	30
6	47

+13, +15, +17

(3,2) (4,15)

Quadratic

$\frac{15-2}{4-3}$

c) $g(x)$ from 3 to 4 13

d) $g(x)$ from 5 to 6 17

$\frac{47-30}{6-5} = \uparrow$

x	h(x)
3	6
4	30
5	150
6	750

×5, ×5

$\frac{y_2 - y_1}{x_2 - x_1}$

Exponential

e) $g(x)$ from 3 to 4 ×5

f) $g(x)$ from 5 to 6 ×5